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Validation and implications of an energy-based bedload transport equation

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ABSTRACT

A recently developed bedload equation (Abrahams & Gao, 2006) has the form $i_b = \omega G^{3\cdot 4}$, where i_b is the immersed bedload transport rate, ω is the stream power per unit area, $G = 1-\theta_c/\theta$, θ is the dimensionless shear stress and θ_c is the associated threshold value for the incipient motion of bed grains. This equation has a parsimonious form and provides good predictions of transport rate in both the saltation and sheetflow regimes (i.e. flows with low and high θ values, respectively). In this study, the equation was validated using data independent of those used for developing it. The data represent bedload of identical sizes transported in various steady, uniform, fully rough and turbulent flows over plane, mobile beds. The equation predicted i_b quite well over five orders of magnitude. This equation was further compared with six classic bedload equations and showed the best performance. Its theoretical significance was subsequently examined in two ways. First, based on collision theory, the parameter G was related to the ratio of grain-to-grain collisions to the total collisions including both grain-to-grain and grain-to-bed collisions, P_g by $P_g = G^2$, suggesting that G characterizes the dynamic processes of bedload transport from the perspective of granular flow, which partly accounts for the good performance of the equation. Moreover, examining the ability of two common equations to predict bedload in gravel-bed rivers revealed that G can also be used to simplify equations for predicting transport capacities in such rivers. Second, a simple dimensionless form of the equation was created by introducing $B = i_b/\omega$. The theoretical nature of the term B was subsequently revealed by comparing this equation with both the Bagnold model and two commonly used parameters representing dimensionless bedload transport rates.

Keywords Bedload transport, stream power, granular flow, gravel-bed rivers.

INTRODUCTION

Intrinsic properties of natural rivers, such as heterogeneous sediment transport, the interaction between sediment supply and bed surface adjustment, and the hydrodynamics of bedform (for example, sand bars) evolution, make the relationship between bedload transport rates and hydraulic variables extremely complex. Although scientists and engineers have gained profound insight into the mechanics of bedload transport ever since the development of the DuBoys equation (DuBoys, 1879), the first physically based bedload transport equation, an apparently simple question still cannot be answered: *For given hydraulic and sedimentary characteristics, what is the rate of bedload transport in an alluvial channel?* In other words, there is no single bedload equation that can be applied *universally* to all rivers (Gomez & Church, 1989; Gomez, 1991; Simons & Senturk, 1992; Yang & Huang, 2001; Almedeij & Diplas, 2003). This lack of universal characterization of bedload transport is caused partly by practical limitations, such as errors in measurement of shear stress and sediment sampling (Gomez & Church, 1989), and is partly due to complexities introduced by heterogeneity of sediment sizes (Parker *et al.*, 1982; Wilcock, 2001) and non-uniform and unsteady flows in natural rivers. The latter includes significant changes of water surface slope during floods (Meirovich *et al.*, 1998; Powell *et al.*, 2006), cross-channel variations of bed shear stress, flow velocity and transport rates (Powell *et al.*, 1999, 2006; Ferguson, 2003) or bedload pulsing due to the phase difference between sediment and water discharges (Reid & Frostick, 1987).

Without these limitations and complexities, can bedload transport be universally characterized by a single equation? A recently developed bedload transport equation (Abrahams & Gao, 2006) for steady, uniform, fully rough and turbulent flows, transporting a bedload of homogeneous grains over plane, mobile beds (hereafter referred to as *ideal flow*) has provided an affirmative answer. This equation was developed based on the Bagnold energy model (Bagnold, 1973):

$$i_b \tan\alpha = e_b \omega \tag{1}$$

where $\tan\alpha$ is the dynamic friction coefficient, e_b is the transport efficiency, $i_b = q_b g(\rho_s - \rho)$ is the immersed bedload transport rate (J s^{-1} m^{-2}), q_b is the volumetric bedload transport rate (m^2 s^{-1}), g is the acceleration of gravity (m s^{-2}), ρ_s is the density of bedload grains (kg m^{-3}), ρ is the flow density (kg m^{-3}), $\omega = \tau u$ is the stream power per unit area (or unit stream power) (W m^{-2}), $\tau = \rho g h S$ is the bed shear stress (N m^{-2}), h is the mean flow depth (m), S is the energy slope and u is the mean flow velocity (m s^{-1}). The dynamic friction coefficient, $\tan\alpha$ is defined as the ratio of the tangential shear stress due to grain collisions, T_g to the associated normal dispersive stress, P, which is equivalent to the submerged weight of bedload grains, W'.

In general, bedload may be transported in either the saltation or sheetflow regimes, which are hydraulically distinguished by $\theta \approx 0{\cdot}5$ (Gao, 2008), where $\theta = \rho h S/((\rho_s - \rho)D_{50})$ is the dimensionless shear stress and D_{50} is the median size of bedload grains (m). In the saltation regime, grains move along the bed by sliding, rolling or saltating with θ values less than 0·5. The total shear stress available for transporting bedload, T, which is equivalent to $t - t_c$, where τ_c is the threshold value of τ for the incipient motion of grains, contains not only T_g but also a component that is transferred to the bed due to friction or turbulence to maintain the equilibrium of the fluid (Graf, 1998), T_f. Thus, $T = t - t_c = T_g + T_f$. In the sheetflow regime where $\theta > 0{\cdot}5$ and grains mainly travel in loosely defined granular layers within a zone that is much thicker than D_{50}, the proportion of T_f becomes smaller and smaller as θ increases. At very high θ values, $T \approx T_g$. Therefore, Eq. 1 is theoretically confined to flows with high θ values in the sheetflow regime.

By replacing $\tan\alpha$ in Eq. 1 with the stress coefficient $s_b = T/W'$, Abrahams & Gao (2006) modified the original energy model as:

$$i_b s_b = e_b \omega \tag{2}$$

and developed equations for both s_b and e_b:

$$s_b = 0{\cdot}6G^{-2} \tag{3a}$$

$$e_b = 0{\cdot}6G^{1{\cdot}4} \tag{3b}$$

A new bedload equation was subsequently established by combining Eq. 2 with Eqs 3a and 3b:

$$i_b = \omega G^{3{\cdot}4} \tag{4}$$

where:

$$G = 1 - \frac{\theta_c}{\theta} = 1 - \frac{u_{*c}^2}{u_*^2} \tag{5}$$

$u_* = (ghS)^{0{\cdot}5}$ is the shear velocity (m s^{-1}), and θ_c and u_{*c} are the threshold values of θ and u_* for the incipient motion of grains on the bed. Equation 4 applies for both the saltation and sheetflow regimes and serves as a universal equation to predict bedload transport rates under any hydraulic and sedimentary conditions in the ideal flows. Although of Eq. 4 is only valid in the ideal flows, its simplicity merits further investigation.

This article presents such an investigation. It begins by validating Eq. 4 using an independent set of bedload data, which is followed by comparing the predictive power of Eq. 4 with that of several classic bedload equations. Then, the theoretical implications of Eq. 4 are revealed by demonstrating the meaning of G in Eq. 4 and illustrating the advantage of a new dimensionless form of Eq. 4, respectively. The article ends with a discussion of the two dimensionless variables in a new dimensionless form.

VALIDATING THE PREDICTABILITY OF EQUATION 4

Although Eq. 4 was tested in Abrahams & Gao (2006), further validating it not only assures its robustness and generality but also justifies the subsequent analysis in this article.

Data compilation

A data set that is for bedload of homogeneous grains and entirely independent of that used by Abrahams & Gao (2006) was compiled to validate Eq. 4. Using this data set avoids the possible errors raised from validating an equation by employing part of the data used to develop it (Gomez & Church, 1989). The validation data set consists of 264 flume or closed-conduit experiments from Recking (2006) and Nnadi & Wilson (1992), and those compiled by Johnson (1943), Smart & Jäggi (1983) and Gomez & Church (1988). To ensure the compatibility of data from different experiments, the sidewall-drag effect was corrected using the method of Williams (1970) except for the data from Nnadi & Wilson (1992) who corrected the original data using their own method. The effect of slope on the bed shear stress was corrected using the same method employed by Abrahams & Gao (2006).

No theoretical method exists to accurately determine values of θ_c (Buffington & Montgomery, 1997). Therefore, for each series of experiments, the value of θ_c was determined by fitting lines through plots of i_b against θ, extending these lines to the θ axis and recording the value of θ, where $i_b = 0$. For experiments where the smallest values of i_b were significantly greater than 0, such as those from Smart & Jäggi (1983) and Nnadi & Wilson (1992), the extrapolation method failed to provide a reasonable estimate of θ_c and thus the value of θ_c was set to equal to 0·04.

The selected data were filtered to only keep those representing turbulent and fully rough flows, which means that the flow Reynolds number $R_h = 4hu/v > 8000$ and the roughness Reynolds number $R_{ks} = k_s u_*/v > 70$ where v is the kinematic viscosity (m^2 s^{-1}) and k_s was determined in the same way as in Abrahams & Gao (2006).

When a grain is entrained in a flow, whether it moves as bedload or goes into suspension depends on the threshold value of the dimensionless settling velocity $W = w/u_*$, where the settling velocity, w, was determined using the equation developed by Cheng (1997a). In the saltation regime, suspension occurs when $W < 1{\cdot}5$, while in the sheetflow regime, the initiation of suspension occurs when $W < 0{\cdot}8$ (Abrahams & Gao, 2006). However, in the sheetflow regime, bedload concentration is significantly higher than that in the saltation regime. Given that w is positively correlated with concentration (Cheng, 1997b), the true value of w should be greater than that calculated using the Cheng (1997a) equation. Inasmuch as suspended concentration begins to increase significantly when $W = 0{\cdot}15$ (Wilson, 2005), the threshold value of W in this study was set as 0·55, between 0·8 and 0·15, to take sediment concentration into consideration. Those measurements with calculated values of W less than 0·55 in the sheetflow regime have significant suspended sediment and were excluded from the validation data set.

Validation

Data from 186 experiments (Table 1) performed in the ideal flows were therefore selected for validating Eq. 4. These data had θ values ranging from

Table 1. Ranges of relevant hydraulic and sedimentary variables of the compiled independent experimental data in the ideal flows.

Sources	Nnadi & Wilson (1992)	Recking (2006)	Johnson (1943)	Smart & Jäggi (1983)	Gomez & Church (1988)	Total
Number of experiments	9	73	42	55	7	186
Re ($\times 10^6$)	0·135–0·256	0·014–0·128	0·039–0·303	0·014–5·04	0·347–0·574	0·014–5·04
Re_*	71–99	86–1001	70–397	95–4639	241–357	70–4639
Fr*	2·53–3·22	1·12–2·58	0·48–1·85	0·42–1·67	0·80–0·95	0·45–3·22
D_{50} ($\times 10^{-3}$ m)	0·7	2·3–9·0	1·7–7·1	5·2–28·7	6·5	0·7–28·7
θ	0·964–1·850	0·091–0·312	0·049–0·260	0·05–0·144	0·034–0·074	0·034–1·850
θ_c	0·04	0·04	0·03–0·053†	0·04	0·029	0·029–0·053
B	0·858–1·168	0·088–0·627	2·9E-04–0·481	1·7E-3–0·190	0·0005–0·0534	2·9E-04–1·168

*Froude number $Fr = u/(gh)^{0{\cdot}5}$.
†These data contain several sets of flume experiments, each of which has a θ_c value.

0·034 to 1·85 and hence covered bedload transport in both the saltation and sheetflow regimes. Both subcritical and supercritical flows were included and values of D_{50} in these data ranged from sand (0·0007 m) to gravel (0·0287 m). Therefore, the compiled data covered a wide range of hydraulic and sedimentary conditions. Comparison of predicted with measured i_b (Fig. 1) showed that data points were generally located symmetrically around the line of perfect agreement over five orders of magnitude. Only less than 9% of the total points were plotted outside of (but still close to) the zone between the discrepancy ratios (van Rijn, 1984; Almedeij & Diplas, 2005; Camenen & Larson, 2005) of 0·5 and 2, which further indicates the general agreement between measured and predicted values of i_b. Given that the selected data are independent of those used in Abrahams & Gao (2006), the good agreement between measured and predicted i_b confirms the general success of Eq. 4 for predicting bedload transport rates of homogeneous sediments in various steady, uniform, fully rough and turbulent flows over plane, mobile beds.

By converting i_b into the well-known dimensionless form $\phi = q_b/(g(\rho_s\text{-}\rho)/\rho D_{50})^{0.5} D_{50}$, first introduced by Einstein (1950), Eq. 4 may be made dimensionless (Abrahams & Gao, 2006):

$$\phi = \theta^{1.5} G^{3.4} \frac{u}{u_*} \tag{6}$$

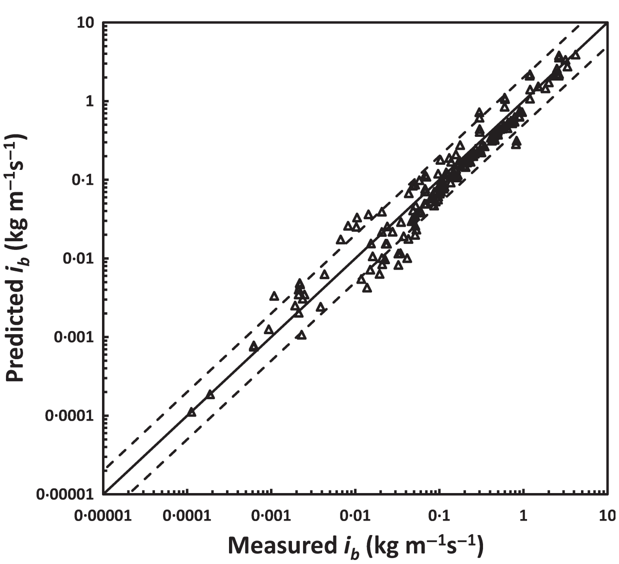

Fig. 1. Predicted i_b using Eq. 4 versus measured i_b. The two dashed lines represent the discrepancy ratios of 0·5 and 2, respectively. The boundary i_b value between the saltation and sheetflow regimes is *ca* 0·65 kg m^{-1} s^{-1}.

The predictive ability of Eq. 4 was subsequently compared with that of six classic and well-known bedload equations developed in terms of excess dimensionless shear stress, $\theta - \theta_c$ (see the first six equations in Table 2). All of these equations can be presented in dimensionless forms similar to Eq. 6. The last two equations in Table 2 were developed for fractional bedload transport over beds of heterogeneous grains and were selected for illustrating a property of G that is elaborated below.

Using the independent data described previously, i_b values predicted by each of the first six equations were compared with the measured transport rates (Fig. 2A to F). The equations of Meyer-Peter and Müller, Shields, Bagnold and Fernandez Luque & van Beek can only predict a small proportion of the data reasonably well (i.e. those falling in the zone between the two discrepancy lines). The equations of both Smart and Yalin can predict medium and high bedload transport rates reasonably well, but generate significantly larger errors in predictions for low bedload transport rates. None of these equations predict bedload transport rates in both the saltation and sheetflow regimes as well as Eq. 4 does.

The results shown in Figs 1 and 2 verify that bedload transport rates for homogeneous sediments in ideal flows can generally be predicted by Eq. 4. Mathematically, the first six equations in Table 2 have a coefficient with different values, suggesting that they can only perform well for flows that have similar hydraulic and sediment conditions to those of the data used to develop them. By contrast, the dimensionless form of Eq. 4 (i.e. Eq. 6) has no coefficient, implying that Eq. 4 reflects the functional relationship between bedload transport rates and the hydraulic variables involved. Some insight may be gained by investigating the theoretical meaning of the variables in Eq. 4.

THEORETICAL SIGNIFICANCE OF EQUATION 4

The parameter *G*

The physical meaning of the dimensionless parameter G in Eq. 4 can be uncovered using the collision theory (Leeder, 1979). In this theory, Leeder (1979) assumed that bedload grains moving in saltation are controlled by grain-to-bed (GB) and grain-to-grain (GG) collisions. Leeder (1979) then derived an expression for the mean

Table 2. Eight selected bedload equations. The first six are based on excess dimensionless shear stress $\theta - \theta_c$, while the last two are based on the transport stage θ/θ_c.

Sources	Bedload equations
Meyer-Peter & Müller (1948)	$\phi = 8(\theta - \theta_c)^{1\cdot5} = 8\theta^{1\cdot5}G^{1\cdot5}$
Fernandez Luque & van Beek (1976)	$\phi = 5{\cdot}7(\theta - \theta_c)^{1\cdot5} = 5{\cdot}7\theta^{1\cdot5}G^{1\cdot5}$
Bagnold [obtained from Yalin (1977)]	$\phi = 4{\cdot}25\theta^{0\cdot5}(\theta - \theta_c) = 4{\cdot}25\theta^{1\cdot5}G$
Shields [obtained from Simons & Senturk (1992)]	$\phi = \frac{10\theta(\theta-\theta_c)(u/s)}{[gD(s-1)]} = \frac{10\theta^2 G(u/s)}{[gD(s-1)]},\ s = \frac{\rho s}{\rho} - 1$
Smart (1984)	$\phi = 4S^{0\cdot6}\frac{u}{u_*}\theta^{0\cdot5}(\theta-\theta_c) = 4S^{0\cdot6}\frac{u}{u_*}\theta^{1\cdot5}G$
Yalin (1977)	$\phi = A\theta^{0\cdot5}(\theta-\theta_c) = A\theta^{1\cdot5}G,$
	$A = \frac{0{\cdot}635}{\theta_c}\left[1 - \frac{ln(1+as)}{as}\right], as = \frac{2{\cdot}45}{S^{0\cdot4}}\theta_c^{0\cdot5}\left(\frac{\theta}{\theta_c} - 1\right)$
Parker *et al.* (1982)	$\phi = 0{\cdot}0025\theta^{1\cdot5}exp\left[14{\cdot}2\left(\frac{\theta}{\theta_c}-1\right) - 9{\cdot}28\left(\frac{\theta}{\theta_c}-1\right)^2\right]\ \frac{\theta}{\theta_c} \leq 1{\cdot}65$
	$\phi = 11{\cdot}2\theta^{1\cdot5}\left(1 - \frac{0{\cdot}822}{\theta/\theta_c}\right)^{4\cdot5}\ \frac{\theta}{\theta_c} \geq 1{\cdot}65$
Wilcock & Crowe (2003)	$\phi = 0{\cdot}002\theta^{1\cdot5}\left(\frac{\theta}{\theta_c}\right)^{7\cdot5}\ \frac{\theta}{\theta_c} < 1{\cdot}35$
	$\phi = 14\theta^{1\cdot5}\left(1 - \frac{0{\cdot}894}{(\theta/\theta_c)^{0\cdot5}}\right)^{4\cdot5}\ \frac{\theta}{\theta_c} \geq 1{\cdot}35$

free path length of a saltating grain, λ, in terms of the gaseous kinetic theory and compared it with the measured mean length of a saltation trajectory, L, to characterize the nature of GG and GB collisions. This expression led to the following outcomes: for $u_*/u_{*c} < 2$, $\lambda > L$ meaning that the transport of bedload grains is dominated by GB collisions; for $u_*/u_{*c} > 2$, $\lambda < L$ suggesting that GG collisions dominate. At $u_*/u_{*c} \approx 2$, $\lambda = L$ signifying that the probability of GB and GG collisions is the same.

Based on the concept of GG and GB collisions, a new variable, P_g (Abrahams & Gao, 2006), was defined as the relative frequency of GG collisions with regard to the total (GG and GB) collisions during bedload transport at a given transport stage, which is commonly defined using u_*/u_{*c} (Leeder, 1979), τ/τ_c (or θ/θ_c) (Wiberg & Smith, 1989). This variable should be negatively related to λ/L but the specific function cannot be derived directly. However, by definition, when $\lambda = L$, $P_g = 0{\cdot}5$; this provides a reference point for constructing a simple function that satisfies the negative trend between P_g and λ/L:

$$P_g = \frac{1}{n+1} \tag{7}$$

where $n = \lambda/L$. Several pairs of u_*/u_{*c} and n were reproduced from the data shown in figure 4B of Leeder (1979). Values of n were subsequently used to calculate P_g (Table 3). Examining the relationship between P_g and a variety of hydraulic variables showed that the data in Table 3 may be best modelled by (Fig. 3):

$$P_g = G^2 \tag{8}$$

Equation 8 signifies that G is directly related to the relative frequency of GG collisions during bedload transport in both the saltation and sheetflow regimes. As θ increases, more grains can be transported as bedload and hence the probability of GG collisions increases, which is quantitatively associated with the increase of G.

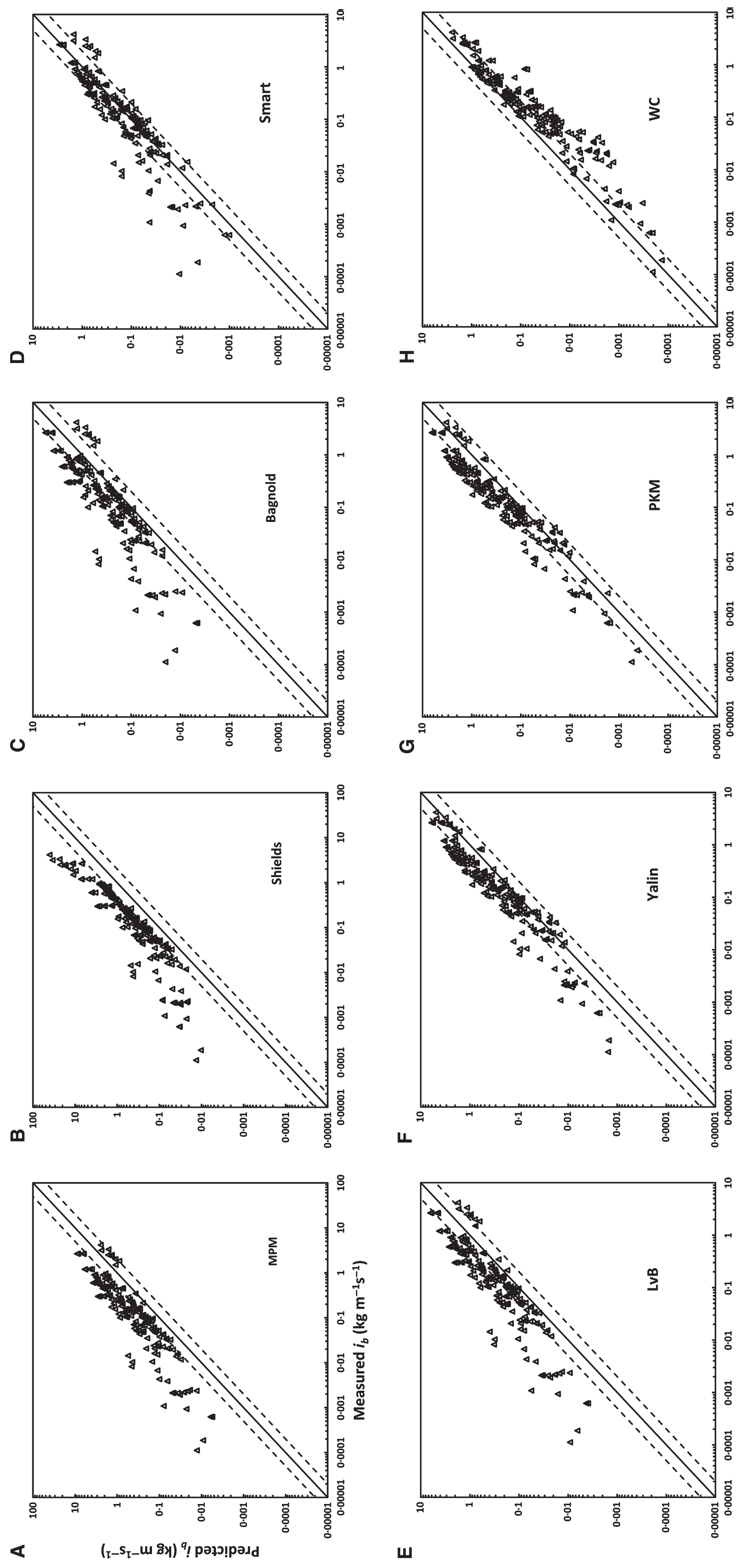

Fig. 2. Comparison of predicted bedload transport rates using the selected eight equations (Table 2) with the measured ones: MPM – Meyer-Peter and Müller; LvB – Fernandez Luque and van Beek; PKM – Parker, Klingeman and Mclean; WC – Wilcock and Crowe.

Table 3. Values of u_*/u_{*c} and P_g obtained from the data in figure 4B of Leeder (1979).

u_*/u_{*c}	P_g
1·67	0·4
1·70	0·455
1·75	0·476
1·85	0·5
2·20	0·625
3·20	0·85

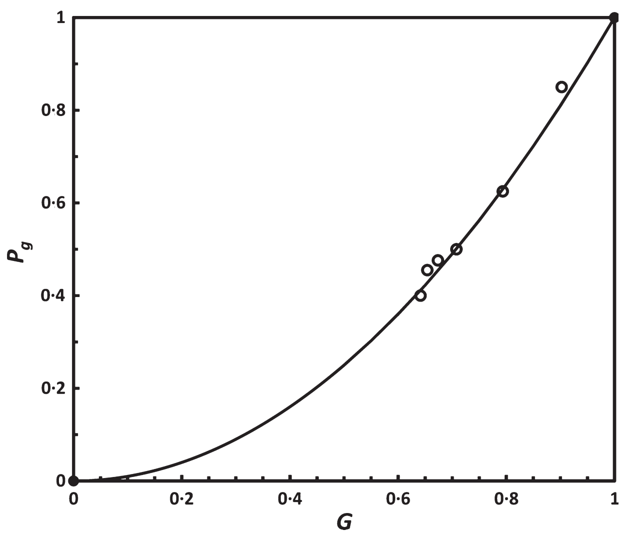

Fig. 3. P_g versus G. The open circles are the data reproduced from the data in figure 4B of Leeder (1979), the solid curve is Eq. 8, and the two solid dots are two extreme points that satisfy Eq. 8 (i.e. the boundary conditions).

Therefore, G is a critical variable that captures the dynamic processes of bedload transport for homogeneous grains over mobile beds when bedload transport is viewed from the perspective of granular flow (i.e. bedload transport is the process of grain movement). It follows that this parameter alone can account for the changes of bedload transport rates with θ values and lead to a simple bedload equation without any coefficient (i.e. Eq. 4 or Eq. 6) that performs well in both the saltation and sheetflow regimes. Although G is also included in the six classic equations (Table 2), it is essentially derived from the commonly used dimensionless excess shear stress, $\theta - \theta_c$ and is not an independent variable as in Eq. 6.

Natural rivers do not generally transport homogeneous sediments and flows are more complex than the ideal flows in the experiments used to define and validate Eq. 4. This model cannot, therefore, be used to predict bedload for heterogeneous sediments in gravel-bed rivers. However, the parameter G still provides insight into the associated transport mechanics. For instance, two well-known bedload equations for bedload of heterogeneous sediments (the last two equations in Table 2) involve two different expressions for estimating bedload transport rates in two different ranges of θ/θ_c values. This suggests that bedload of finer sediments for low θ/θ_c values follows different hydraulic rules from that of coarse sediments for high θ/θ_c values. However, when the two equations were applied to the independent data set for predicting bedload of identical sediments, the trends of the predicted i_b values are approximately parallel to the line of equality (see Fig. 2G and H), although neither of them predicts bedload transport rates as well as Eq. 4 does. This implies that use of G for bedload of heterogeneous grains might enable collapse of the two different expressions for the two different ranges of θ/θ_c values. Following from this, a bedload equation that predicts bedload transport capacity of heterogeneous sediments in gravel-bed rivers has been developed recently using only one independent variable, G (Gao, 2011). The utility of G for characterizing the mechanics of bedload transport provides further evidence to support the emerging assertion that bedload transport shares many common properties with granular flows, such that more insight may be gained if bedload transport is treated as the process of granular interaction/collision (Frey & Church, 2011).

A different dimensionless form of Equation 4

Equation 4 indicates that as shear stress increases, submerged bedload transport rate, i_b increases faster than the unit stream power, ω. At very high shear stress, the bedload transport rate, i_b numerically approaches ω. This finding is at odds with the Bagnold (1973) model. Rearranging Eq. 1 gives:

$$i_b = \frac{e_b}{\tan\alpha}\omega = 1{\cdot}67 e_b \omega \tag{9}$$

where $\tan\alpha \approx 0{\cdot}6$. Bagnold (1973) asserted that when the transport efficiency reaches its maximum, e_b = 100% and i_b/ω = 1·67. In other words, i_b can be numerically greater than ω. The source of this errant assertion is that the transport efficiency can never reach 100%. Rather, its maximum value is 60% (see Eq. 3b and Abrahams & Gao, 2006). Therefore, the classic representation of a range of e_b values (up to 100%) in the plot of i_b versus ω as a series of curves (Emmett, 1976; Leopold & Emmett, 1976; Reid & Laronne, 1995; Knighton, 1998) should be modified.

It follows that ω not only represents the total energy a flow has (i.e. the ability of doing work) but also serves as a numerical scale that can normalize i_b – that is, i_b in open-channel flows cannot be greater than ω. Thus, a different dimensionless form than Eq. 6 may be easily generated as:

$$B = \frac{i_b}{\omega} = G^{3 \cdot 4} = \left(1 - \frac{\theta_c}{\theta}\right)^{3 \cdot 4} \tag{10}$$

In addition to its simplicity, Eq. 10 also has a theoretical advantage that may be revealed by comparing B with the two widely used parameters representing dimensionless bedload transport rates. The first is ϕ and the second is W^*, which is defined as $W^* = (\rho_s/\rho - 1)gq_b/u_*^3$ (Parker & Klingeman, 1982). Their mathematical relationships are:

$$B = \frac{i_b}{\omega} = W * \frac{u_*}{u} = \frac{\phi}{\theta^{1 \cdot 5}} \frac{u_*}{u} \tag{11}$$

Given that $\frac{u_*}{u} = \sqrt{\frac{f}{8}}$ where f is Darcy–Weisbach friction factor, Eq. 10 suggests that B incorporates the independent impact of resistance to flow on bedload transport rates into both ϕ and W^*. Because f may vary greatly with G in both the saltation and sheetflow regimes (Fig. 4), use of B circumvents the need for additional variables or constants to account for such variation and further explains the generality of Eq. 10.

The implication of Equation 10

The hydraulic implication of Eq. 10 can be revealed by the following reasoning, based on the assumption that bedload grains always have the same size and, hence, θ_c remains invariable.

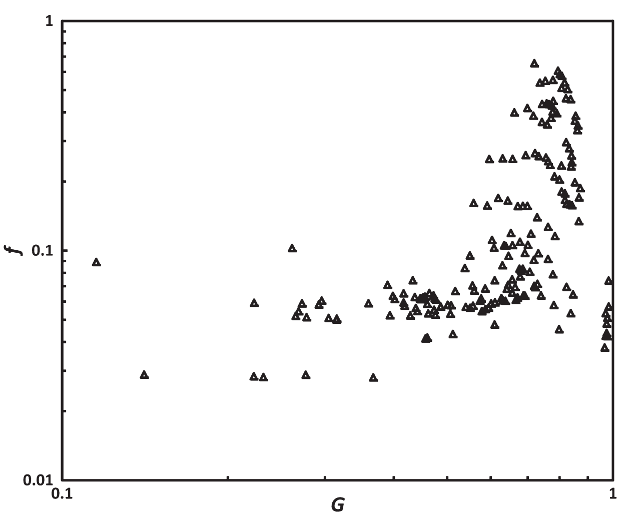

Fig. 4. Variations of resistance to flow, f, with G in both the saltation and sheetflow regimes.

With this assumption, the magnitude of i_b could be controlled by θ, ω or both. When two flows have the same θ value, meaning the same product of flow depth (h) and energy slope (S), one flow may have higher velocity and unit discharge, which leads to a higher stream power than the other. This effect can be achieved by having lower flow resistance in the first flow. Thus the flow with higher ω will transport more bedload (i.e. higher i_b). When two flows have the same ω value, the product of unit discharge (Q) and energy slope (S) is the same in the two flows. If the first flow has a high enough Q and relatively gentle bed slope with relatively narrow channel width, then it is possible that this flow has a higher shear stress, although the associated flow velocity may be low and flow resistance may be high. Therefore, the higher θ value will give rise to a higher i_b. These two cases indicate that either θ or ω may vary, as one of them remains constant and, consequently, causes a change of bedload transport rates; this suggests that neither ω nor θ controls bedload transport alone. Physically, Eq. 10 reveals that the proportion of bedload transport rate to stream power (i.e. B) is correlated to the relative frequency of GG collisions with respect to the total collisions, which can be determined by G. Numerically, both B and G vary from 0 to 1. Thus, Eq. 10, which is simple and general, is determined by both G and B. It suggests that under simplified, ideal flow conditions, the mechanics of bedload transport is remarkably simple: any combination of ω and i_b leads to a unique B that can be determined simply using the associated G. In natural gravel-bed rivers where bedload sediments and those from both the bed surface and substrate are heterogeneous, bedload transport is more complex than described by Eq. 10. Thus, Eq. 10 cannot be used directly to estimate bedload transport rates in natural rivers. However, B and G can still be used to provide some quantitative benchmarks for more complex investigation of bedload transport, as demonstrated in the equation for bedload transport capacities of gravel-bed rivers established in Gao (2011).

CONCLUSIONS

A data set was carefully compiled to validate, in two ways, a previously published equation for predicting bedload under ideal conditions (steady, uniform, fully rough and turbulent flows transporting homogeneous grains over plane, mobile beds) (Equation 4; Abrahams & Gao, 2006). First, predicted i_b values were compared with measured values and showed good agreement. Second, six classic bedload equations were selected and compared with Eq. 4 using the compiled data set. The results showed that Eq. 4 has the best predictive power. Thus, Eq. 4 is a general equation that can predict bedload transport rates under a wide range of uniform, fully rough and turbulent flows transporting bedload of homogeneous grains over plane, mobile beds. Its form has some significant implications regarding the mechanics of bedload transport.

The parameter G is found to be related to the proportion of grain-to-grain collisions to the total [the sum of grain-to-grain (GG) and grain-to-bed (GB)] collisions, P_g by $P_g = G^2$. This relation suggests that if bedload is viewed from the perspective of granular movement, changes in bedload transport rates with hydraulics are essentially controlled by the change of the relative frequency of GG and GB collisions, which is characterized by G. Although flows in gravel-bed rivers are more complex than the ideal flows, G enables a collapse of two different expressions associated with the two different ranges of θ/θ_c in two bedload equations for gravel-bed rivers (Fig. 2G and H) and can be used to determine bedload transport capacities of heterogeneous grains in gravel-bed rivers (Gao, 2011). Therefore, the parameter G is a fundamental hydraulic variable that captures the processes of bedload transport.

Moving ω in Eq. 4 to the left gives rise to a much simpler dimensionless form (i.e. Eq. 10). Comparing the term B with the two commonly used parameters for dimensionless bedload transport rate, ϕ and W^*, showed that B accounts for the effect of the change of resistance to flow on bedload transport and thus has the hydraulic advantage of merging data into a single relation. The dimensionless bedload transport rate, B in Eq. 10 is fundamentally different from i_b/ω used by Bagnold (1973) in that the latter can be greater than 1 for high θ values. Numerically, B ranges from 0 to 1, suggesting that ω can normalize i_b such that the change of B is consistent with the change of the relative frequency of GG and GB collisions, and hence B can be simply determined using G.

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