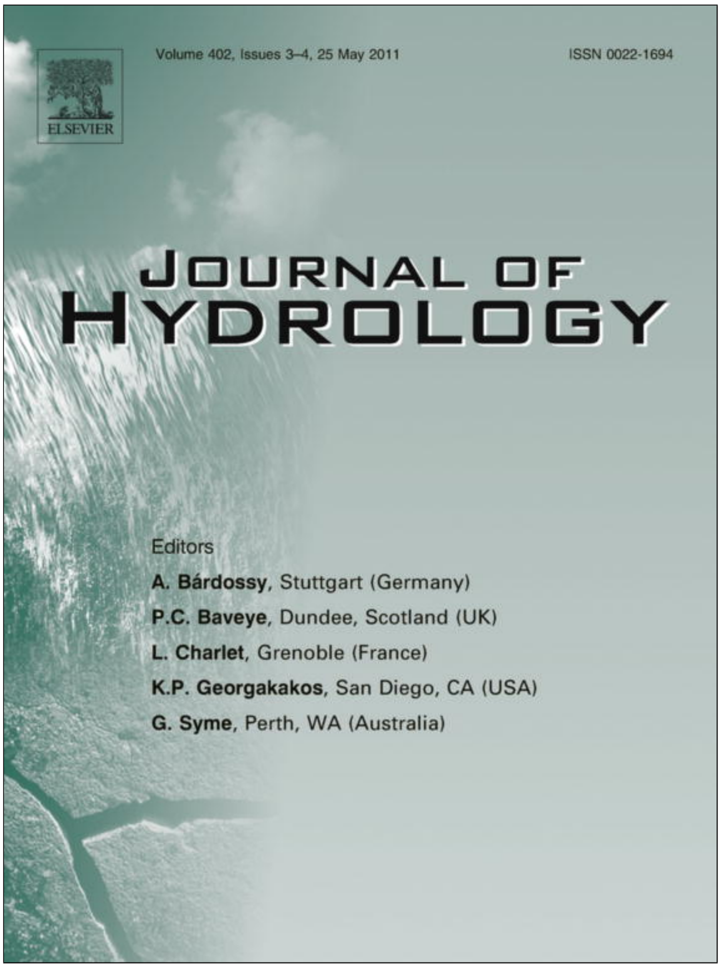
Volume 402, Issues 3–4, 25 May 2011
ISSN 0022-1694
ELSEVIER
JOURNAL OF HYDROLOGY
Editors
A. Bárdossy, Stuttgart (Germany)
P.C. Baveye, Dundee, Scotland (UK)
L. Charlet, Grenoble (France)
K.P. Georgakakos, San Diego, CA (USA)
G. Syme, Perth, WA (Australia)

Journal of Hydrology 402 (2011) 297–305

Contents lists available at ScienceDirect

Journal of Hydrology

journal homepage: www.elsevier.com/locate/jhydrol

An equation for bed-load transport capacities in gravel-bed rivers

Peng Gao *

Department of Geography, Syracuse University, Syracuse, NY 13244, United States

ARTICLE INFO

Article history:
Received 16 September 2010
Received in revised form 6 March 2011
Accepted 19 March 2011
Available online 12 April 2011
This manuscript was handled by K. Georgakakos, Editor-in-Chief, with the assistance of Ehab A. Meselhe, Associate Editor

Keywords:
Bed-load transport
Transport capacity
Armor layer
Gravel-bed rivers

SUMMARY

Detailed hydraulic and sedimentary information is needed to accurately predict bed-load transport rates in natural gravel-bed rivers. Yet, being able to estimate maximum transport rates from simple measurements would greatly benefit various sediment-related river management practices. To this end, a new concept of bed-load transport capacity for heterogeneous grains in gravel-bed rivers was introduced as the maximum possible transport rate a gravel-bed river can have for a given value of dimensionless shear stress, calculated using the median size of bed-load grains. Flows that can transport bed load at capacity may be identified by the criterion that the median size of bed-load grains must be greater than or equal to that of the bed substrate. Then, a single coefficient, power equation was developed to predict such capacities using bed-load capacity data covering both low flows with an armor layer and high flows without it. The good performance of this empirical equation was confirmed by comparing its predictability with that of Mayer Peter and Muller's and Bagnold's bed-load equations. Using an independent data compiled from six gravel-bed rivers in Idaho, not only was the empirical equation validated but also the criterion for identifying the condition under which bed load is transported at capacity was tested. In practice, the empirical equation can be used to estimate the maximum possible bed-load transport rates during high flow events, which is useful for various sediment-related river managements.

1. Introduction

Estimating bed-load transport rates in natural gravel-bed rivers without collecting large amounts of detailed data on river hydraulics, bed material size distribution, and channel bedforms is very difficult and remains a major problem with respect to managing rivers for both ecosystem functions and navigation (Bunte et al., 2008). The complexity of determining bed-load transport rates in natural gravel-bed rivers mainly stems from: (a) variable upstream sediment supply (e.g., Dietrich et al., 1989), (b) unsteady flows that may result in cross-channel variations of bed shear stress, flow velocity, and transport rates (e.g., Ferguson, 2003; Powell et al., 2006, 1999), and (c) heterogeneous sizes of grains both on the bed surface and in the bed substrate. The heterogeneous grains are mostly responsible for the renowned hydraulic phenomena in gravel-bed rivers: (i) hiding effect and bed armoring (i.e., bigger grains on bed surface preventing smaller ones beneath from being transported and forming a coarser bed surface layer) (e.g., Andrews and Parker, 1987; Egiazaroff, 1965; Einstein and Chien, 1953; Gomez, 1983; Lisle and Madej, 1992; Montgomery et al., 2000; Sutherland, 1987), (ii) selective transport (i.e., grains of different sizes on the bed being transported by different flow intensities) (e.g., Bridge, 2003; Buffington and Montgomery, 1999; Dietrich et al., 1989; Wathen et al., 1995), and (iii) equal mobility (i.e., grains of different sizes may be transported by the same flow intensity) (e.g., Lenzi et al., 1999; Parker et al., 1982; Parker and Toro-Escobar, 2002).

In the past three decades, research done to predict bed-load transport rates in gravel-bed rivers has focused on how bed surface texture and grain sizes vary in response to the change of sediment supply (e.g., Buffington and Montgomery, 1999; Dietrich et al., 1989) and how vertical grain exchange among bed load, bed surface, and substrate happens during transport (e.g., Wilcock, 2001). These efforts have led to many equations that predict bed-load transport rates if detailed information on flow hydraulics, bed surface and substrate characteristics and size distribution, and bed-load grain composition are available (e.g., Parker et al., 1982; Recking, 2010; Wilcock and Kenworthy, 2002). In practice, however, no equation can be universally applied to all rivers (e.g., Barry et al., 2004; Bathurst et al., 1987; Reid et al., 1996) and accurate prediction of bed-load transport rates necessitates a try-and-error procedure to identify the best suitable equation (e.g., Wilcock et al., 2009), as well as the above-mentioned detailed information. Consequently, predicting bed-load transport rates requires considerable effort even in a small stream.

In this paper, a simple bed-load equation that requires simple measurements is developed. This equation is not for estimating specific bed-load transport rates, but the maximum bed-load transport rates (i.e., the transport capacities) of heterogeneous

* Tel.: +1 315 443 3679; fax: +1 315 443 4227.
E-mail address: pegao@maxwell.syr.edu

0022-1694/$ - see front matter © 2011 Elsevier B.V. All rights reserved.
doi:10.1016/j.jhydrol.2011.03.025

grains for given hydraulic and sedimentary conditions in gravel-bed rivers. The paper begins with the definition of transport capacity for heterogeneous grains in gravel-bed rivers. Then, a criterion of identifying such capacities is introduced. A simple bed-load equation is subsequently developed using nonlinear regression based on the compiled data. Both the equation and the criterion are further validated using an independent data from natural gravel-bed rivers. The paper ends with the discussion of the limitations and application of the developed equation.

2. Methods

2.1. Definition of transport capacity

Bed-load transport capacity traditionally refers to "the maximum load of a given kind of debris a stream can carry" (Gilbert, 1914). Engineers and scientists have used the concept of bed-load transport capacity to assess degradation and aggradation rates on river channel beds and to understand if sediment transport is primarily controlled by sediment supply (i.e., supply-limited rivers) or river flow hydrodynamics (i.e., transport-limited rivers) (e.g., Andrew, 1979; Hicks and Gomez, 2003; Jackson and Beschta, 1982; Lisle, 2007; Mackin, 1948; Reid and Dunne, 1996; Sear, 1996). Considerable work to determine what controls bed-load transport capacity has derived from flume experiments that used grains with identical or nearly identical particle size (e.g., Fernandez Luque and van Beek, 1976; Simons and Senturk, 1992; Yalin, 1977), which may be called homogeneous grains. However, bed-load transport in natural gravel-bed rivers never occurs with homogeneous grains, but rather, with grains of mixed sizes, termed heterogeneous grains. Because bed-load transport with homogeneous grains is significantly different from that with heterogeneous grains, the concept of bed-load transport capacity may in fact be different with respect to homogenous and heterogeneous grains.

In a steady, uniform flow transporting homogeneous grains over plane, loose beds, bed-load transport rate is indeed the only transport rate the flow has and can be defined as the transport capacity for homogeneous grains. This capacity has been generally quantified by Abrahams and Gao (2006) using an equation, which can be expressed in a different dimensionless form than used in Abrahams and Gao (2006)

$$B = G^{3.4} \tag{1}$$

where $B = i_b/\omega$, i_b is the bed-load transport rate at capacity (kg m^{-1} s^{-1}), $\omega = \tau u = \rho g h S u$ is the unit stream power per unit bed area (kg m^{-1} s^{-1}) in which τ is the bed shear stress (kg m^{-2}), h is the mean flow depth (m), S is the energy slope, ρ is the density of flow (kg m^{-3}), g is the acceleration of gravity (m s^{-2}), and u is the mean flow velocity (m s^{-1}). The variable G equals to $1 - \theta_c/\theta$ where in $\theta = \rho h S/(\rho_s - \rho)D_{50}$ is the dimensionless shear stress, ρ_s is the density of sediment (kg m^{-3}), D_{50} is the median size of bed-load grains (m), and θ_c is the critical value of θ for the initial movement of sediment. This concept of transport capacity mainly applies to flume experiments with homogeneous grains, though it could also be applicable in sand-bed rivers within a very narrow range of hydraulic conditions (Simons and Senturk, 1992).

In natural gravel-bed rivers containing heterogeneous grains both on the bed surface and in the bed substrate, bed-load transport rates are typically limited by an armor layer developed on the bed surface and hence are lower than those predicted using Eq. (1) for the same θ values. When flow rates are high, which means corresponding θ values calculated based on the median size of bed surface grains, D_{s50}, are roughly above the range of 0.1–0.2 (Ashworth and Ferguson, 1989; Lisle and Smith, 2003; Parker and Klingeman, 1982; Wilcock and Southard, 1989), the armor layer may break out and bed-load transport can occur at capacity (Gomez, 2006; Laronne et al., 1994; Parker, 2006; Powell et al., 1999, 2001; Wilcock and Crowe, 2003). In practice, the breakout of the armor layer most possibly occurs with peak discharges (i.e., big flows) (e.g., Clayton and Pitlick, 2008; Wilcock and DeTemple, 2005) during which bed-load measurement is very difficult to deploy. Thus, there are not many data representing transport capacities in natural gravel-bed rivers available. The widely accepted capacity data are those collected from a natural gravel-bed river, the Nahal Yatir River of Israel (Reid et al., 1995). In this desert ephemeral river, intensive storm events and sufficient bed materials assure that bed load is transported along the bed without an armor layer (Laronne et al., 1994; Powell et al., 2001) and both bed-load transport rate and efficiency are high. Hence, equal mobility is achieved and fractional bed-load transport rates should be described by the same transport equation (Parker, 2006; Powell et al., 2001; Wilcock and Crowe, 2003). In other words, bed load in this river is transported at capacity. If this capacity is conceptually the same as the one predicted by Eq. (1) for homogeneous grains, then the bed-load data collected from the Nahal Yatir River should be predicted reasonably well by Eq. (1).

Based on the data compiled from Reid et al. (1995), bed-load transport rates in Nahal Yatir river were predicted using Eq. (1) and compared with the measured ones. Fig. 1 showed that bed-load transport rates in gravel-bed rivers without the armor layer were consistently lower than the transport capacities predicted using Eq. (1). It follows that bed-load transport rates in gravel-bed rivers with and without an armor layer are generally less than those predicted using Eq. (1). This suggests that the transport capacity in natural gravel-bed rivers transporting bed-load grains of mixed sizes is different from that of homogeneous sizes and needs to be defined separately. Therefore, *bed-load transport capacity for heterogeneous grains is herein defined as the maximum possible transport rate a gravel-bed river can have for a given value of θ calculated using the median size of bed-load grains D_{50}*. According to this definition, though a gravel-bed river with an armor layer may have several different available transport rates for a given θ value, it only has one maximum possible transport rate (i.e., the transport capacity).

2.2. Data compilation and analysis

To develop an equation for bed-load transport capacities in natural gravel-bed rivers, data representing the transport capacities

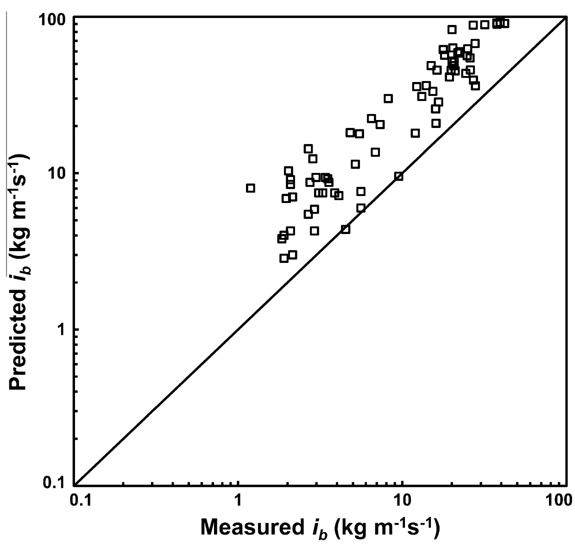

Fig. 1. Comparison of bed-load transport rates in Nahal Yatir River with the transport capacities predicted by Eq. (1) for the same θ values. This gravel-bed river transports bed load of heterogeneous grains without an armor layer and at capacity (Laronne et al., 1994).

for heterogeneous grains are required. Three data sets were compiled from various gravel-bed rivers.

The first data set was from the Nahal Yatir River where high flows transported bed load at capacity without an armor layer. The original data that contained 74 data points were collected every 1 or 2 min for four events (Reid et al., 1995). To reduce the errors due to flow fluctuation, the original data collected every three–five consecutive minutes during each event were averaged, which ended up with 18 data points (Table 1).

The second data set represented bed-load transport in relatively low flows of 22 perennial gravel-bed rivers within the watersheds of diverse land use and sources of bed material in USA (Bravo-Espinosa, 1999). Detailed information can be found in Bravo-Espinosa et al. (2003). The original data had 1015 data points that involved a variety of bed-load transport rates and bed material sizes. Only 25 out of 1015 were finally selected as the qualified data using a two-step procedure. The first step may be described as follow:

- First, data representing turbulent, fully rough flows were selected from the original 1015 data points using the criteria adopted in Abrahams and Gao (2006) – that is the flow Reynolds number $R_h = 4hu\nu > 8000$ and the roughness Reynolds number $R_{ks} = k_s u_*/\nu > 70$, where ν is the kinematic viscosity ($m^2\ s^{-1}$), $u_* = (ghS)^{0.5}$ is the shear velocity ($m\ s^{-1}$), $k_s = mD_{s50}$ is the equivalent sand roughness (m), and m is a coefficient which is assumed to be 3. Because the equation to be developed for bed-load transport capacities for heterogeneous grains will adopt the dimensionless variables used in Eq. (1), which was developed for turbulent, fully rough flows, constraining the data to such flows avoids unnecessary errors due to data from different types of flows.
- Second, in the selected turbulent, fully rough flows, some may contain a significant proportion of suspended load. These data need to be eliminated. The conventional method of judging whether a sediment-laden flow mainly transport bed load or not is based on the dimensionless settling velocity w/u_*, where w is the mean settling velocity of the grains ($m\ s^{-1}$). In the saltation regime, which means flows that satisfy $\theta \leqslant 0.5$ (Gao, 2008), flows that have w/u_* less than about 1.2 transport significant suspended load (Bridge, 2003) and were excluded. In the sheetflow regime, which means flows that satisfy $\theta > 0.5$ (Gao, 2008), flows with w/u_* less than about 0.8 have significant suspended load (Abrahams and Gao, 2006) and were thus removed.

This step gave rise to the total of 329 data points selected from the original 1015 representing turbulent, fully rough flows predominately transporting bed load in natural gravel-bed rivers.

Bearing in mind that the selected data will be used to develop an equation for bed-load transport capacity for heterogeneous grains, it is thus necessary to examine whether all of these 329 data have bed load transported at capacity, which is the task of the second step. According to the previously described definition, transport capacities for heterogeneous grains should be technically reflected by the data points located on top of a plot of B against θ. This is because among all possible bed-load transport rates for a given θ value, the transport capacity refers to the maximum one. Using this technique, the 329 data were displayed in the plot of B against θ. For a given θ value, transport capacity is represented by the data point that has the largest value of B. The capacity data selected in this manner were those within or close to the zone bounded by the two curves in Fig. 2, which led to 25 data points. These selected data constitute the second data set (Table 1) representing flows transporting predominately bed load at capacity from three gravel-bed rivers, Clearwater river, Oak Creek, and Toutle river (Table 1). Although the data number is reduced significantly from 1015 to 25, it is the quality rather than the quantity that is critical for the development of a bed-load equation (Gomez and Church, 1989). The small number of the selected data also indicates that based on the currently defined concept of transport capacity for heterogeneous grains, the transport capacity does not occur very often in natural gravel-bed rivers.

To increase the size of the final data, the third data set was compiled based on the data compiled by Gomez and Church (1988) from several gravel-bed rivers. These data can be simply divided into two groups: those satisfying

$$D_{50} \geqslant D_{sub50} \tag{2}$$

where D_{sub50} is the median size of the bed substrate, and those do not. Examining the data from the first and second data sets revealed that all of these data satisfy Eq. (2). So, the data satisfying Eq. (2) are assumed to represent transport capacities for heterogeneous grains. Using Eq. (2), as well as the criteria for turbulent, fully rough flows and the dominance of bed load in flows, 33 data from Gomez and Church (1988)' s data were selected representing bed-load transport capacities in two gravel-bed rivers, Elbow and Tanana Rivers. These data constitute the third data set (Table 1). The three data sets amount to 76 data standing for bed-load transport capacities for heterogeneous grains in natural gravel-bed rivers. The ranges of values of the key hydraulic and sedimentary variables were shown in Table 1. The value of θ_c for each data set was determined as the value of θ when bed-load transport rates were extrapolated to zero in the plot of i_b against θ (Buffington and Montgomery, 1997). Although the number of qualified data is not large, these data represent transport capacities in natural gravel-bed rivers having a wide range of flow rates with and without the armor layer. They were used to develop an empirical equation for bed-load transport capacities in gravel-bed rivers.

2.3. Equation development and validation

Simple power functions have successfully predicted bed-load transport rates in natural gravel-bed rivers when great details are known (e.g., Barry et al., 2004; Martin, 2003). Thus, in

Table 1
Ranges of relevant hydraulic and sedimentary variables of the compiled data sets reflecting bed-load transport capacities for heterogeneous grains in natural gravel-bed rivers.

Sources	The first data set		The second data set		The third data set		Total
	Nahal Yatir	Clearwater	Oak Creek	Toutle	Elbow	Tanana	
Number of field data	18	3	5	17	19	14	76
Re ($\times 10^6$)	0.06–0.78	10.07–13.19	0.44–0.57	0.58–2.52	0.63–1.25	1.77–3.26	0.06–13.19
Re_*	357–779	3331–3902	2178–2904	262–3462	3598–4134	431–578	357–4134
Fr	0.88–1.24	0.38–0.42	0.80–0.99	0.58–0.90	0.65–0.83	0.25–0.39	0.25–1.24
D_{50} ($\times 10^{-3}$ m)	6	32	18.4–23.3	3.7–19.6	27	7.6	6–32
θ	0.094–0.451	0.054–0.074	0.104–0.126	0.124–0.375	0.105–0.140	0.068–0.122	0.054–0.451
θ_c	0.04	0.033	0.06	0.04	0.052	0.038	0.033–0.06
B	0.185–0.707	0.009–0.020	0.008–0.017	0.148–0.479	0.005–0.077	0.018–0.132	0.005–0.707

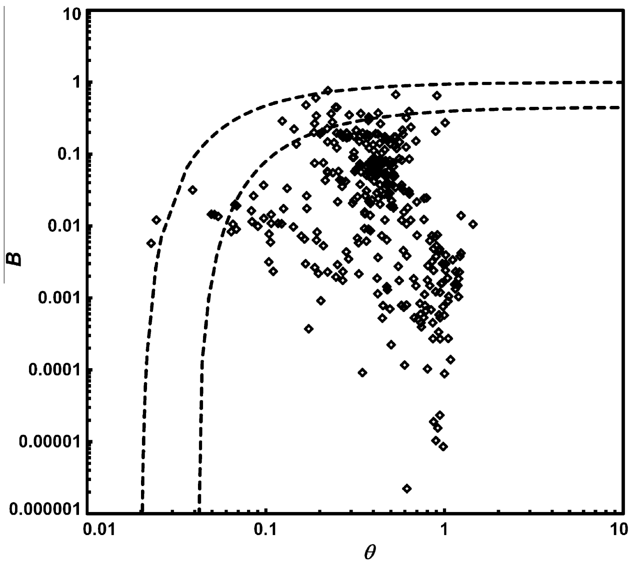

Fig. 2. Identification of the data in the second data set representing bed-load transport capacities for heterogeneous grains. The two dashed curves represent Eq. (1) with $\theta_c = 0.04$ and $\theta_c = 0.02$, respectively.

developing an equation for heterogeneous bed-load transport capacities, a simple power function, similar to Eq. (1), is chosen:

$$B = aG^b \quad (3)$$

where a and b are two coefficients. Values of a and b were determined by nonlinear regression using values of B and G in the selected data (Table 1). Since the established empirical equation will be used to predict bed-load transport capacities, its predictability was examined by a common method that defines the discrepancy ratio as the ratio of predicted to measured bed-load transport rates and calculates the percentage of data points that falls in the zone bounded by discrepancy ratios of 0.5 and 2 (e.g., Almedeij and Diplas, 2003; van Rijn, 1984), respectively.

To show the good predictability of the developed empirical equation for the bed-load transport capacities in natural gravel-bed rivers, two classic and representative bed-load equations were selected for comparison. One is Meyer-Peter and Mueller (1948)'s equation and the other is Bagnold's equation (obtained from Yalin (1977)):

$$\text{Meyer-Peter and Muller} \quad \phi = 8(\theta - \theta_c)^{1.5} \quad (4a)$$

$$\text{Bagnold} \quad \phi = 4.25\theta^{0.5}(\theta - \theta_c) \quad (4b)$$

where the dimensionless bed-load transport rate $\phi = q_b/((\rho_s/\rho - 1)gD_{50}^3)^{0.5}$ (Einstein, 1950), and q_b is the volumetric bed-load transport rate ($m^2\ s^{-1}$).

The empirical equation and the robustness of the criterion that identifies transport capacities (i.e., Eq. (2)) were further tested using an independent data set that contains 557 data points compiled from six gravel-bed rivers in Idaho, USA. These data were downloaded from http://www.fs.fed.us/rm/boise/research/watershed/BAT/index.shtml. In each of these rivers, three to four bed substrate cores were collected. These data were used to calculate the median size of bed substrate D_{sub50}. For each measured water discharge Q, bed-load transport rate and median size of bed-load grains, D_{50}, were measured based on the associated bed-load sample. The corresponding flow width was also measured. The mean flow velocity, u, for the same Q can be calculated using the developed hydraulic geometry (i.e., the u–Q relationship). These values allowed for the calculation of all required hydraulic and sedimentary parameters in the developed equation. The value of θ_c for each data set was determined the same way described previously. These data were subsequently used to validate the developed empirical equation, test the robustness of Eq. (2), and discuss the influence of the armor layer on bed-load transport.

3. Results and discussion

3.1. Equation development and its predictability

Using the data summarized in Table 1, values of a and b in Eq. (3) were determined by regressing B against G: $a = 0.9$ and $b = 6$ with $R^2 = 0.89$ ($p \ll 0.01$). Thus, the empirical equation that predicts bed-load transport capacities for heterogeneous sediments is

$$B = 0.9G^6 \quad (5)$$

Given that both B and G contain a common parameter, bed shear stress τ, the high R^2 value may be partially affected by the possible spurious correlation between the two variables. To avoid such spurious correlation, the predictability of Eq. (5) was further examined by comparing the predicted with measured i_b (Fig. 3). More than 92% of the total data points were within or on the zone marked by the discrepancy ratios of 0.5 and 2 (Fig. 3) indicating that Eq. (5) fits the data reasonably well. Because the data in Table 1 cover from low flows with an armor layer to high flows without it and have bed-load grains ranging in size (i.e., D_{50}) from 6 to 32 mm, Eq. (5) is applicable for predicting bed-load transport capacities in almost all conditions that may occur in natural gravel-bed rivers. Specifically, Eq. (5) may be used in gravel-bed rivers that have turbulent, fully rough flows transporting bed load over plane beds with and without the armor layer to predict transport capacities of heterogeneous grains.

The same data in Table 1 were also used to compare the predictability of the two well-known equations for bed-load transport capacities (i.e., Eqs. (4a) and (4b)) with that of Eq. (5). The results (Fig. 4) showed that both equations tend to over-predict the capacities. Although it is impossible to test all available bed-load equations, the poor predictability of these two commonly used equations suggests that bed-load transport capacities for heterogeneous grains can be best determined by Eq. (5). In this equation, θ

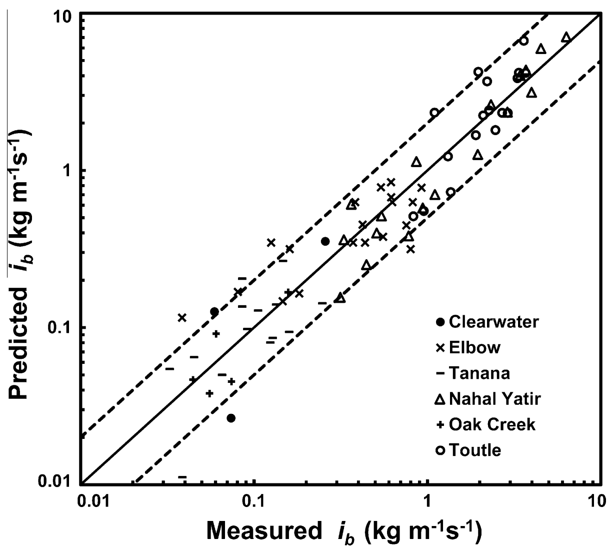

Fig. 3. Comparison of measured bed-load transport capacities for mixed grains with those predicted by Eq. (5). The two dashed lines represent the discrepancy ratios of 0.5 and 2, respectively.

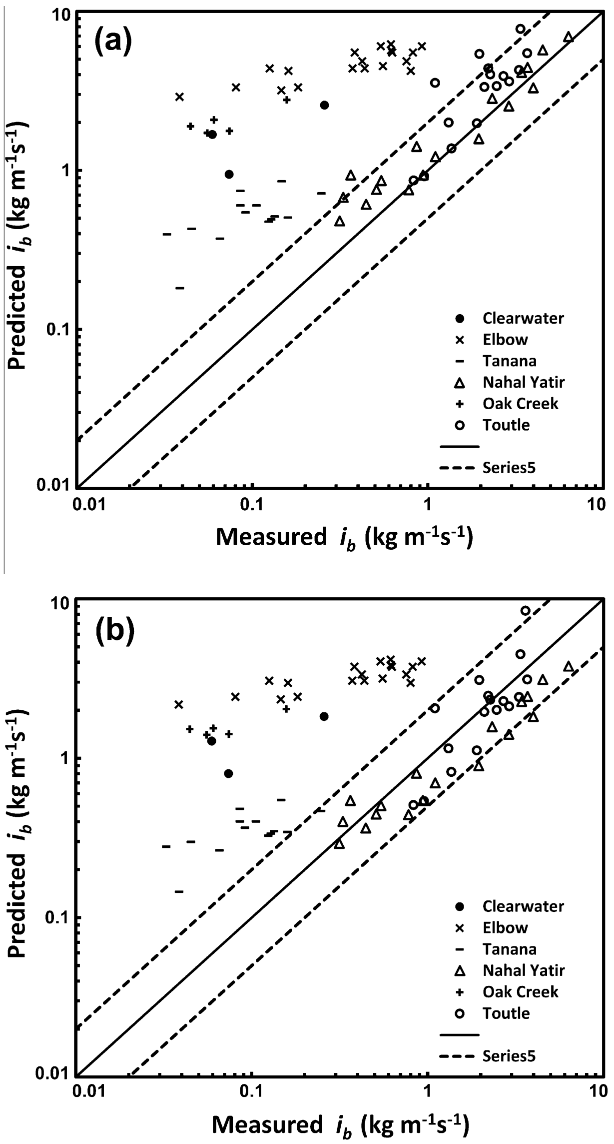

Fig. 4. Comparison of predicted transport capacities using the two selected classic equations with the measured ones. (a) the results based on Meyer-Peter and Muller's equation (i.e., Eq. (4a)); (b) the results based on Bagnold's equation (i.e., Eq. (4b)). The two dashed lines represent the discrepancy ratios of 0.5 and 2, respectively.

is determined using the median size of bed-load grains D_{50}. Thus, Eq. (5) is conceptually different from those developed based on some representative sizes of bed surface grains (e.g., Parker, 1990; Recking, 2010).

3.2. Equation validation

To validate Eq. (5), data representing bed-load transport capacities for heterogeneous grains are required. The independent data compiled from the six gravel-bed rivers in Idaho thus were divided into three groups. The first group contained the data that satisfy Eq. (2) meaning these data representing the transport capacities. These data can be used to directly validate Eq. (5). The second group included the data that do not satisfy Eq. (2) but have D_{50} close to D_{sub50} suggesting their transport rates were below capacities. Therefore, the robustness of Eq. (2) may be validated if their transport rates are indeed less than those predicted by Eq. (5). Details of the data from these two groups were shown in Table 2. The third group consisted of the data that have D_{50} much less than D_{sub50} indicating that their transport rates should be well below the capacities for the same θ values. These data will be primarily used in the following section for discussing the impact of the armor layer on bed-load transport. All data in three groups were then displayed in Fig. 5 where the open triangles denote the data from the first group, the open squares signify the data from the second group, the solid dots symbolize the data from the third group, and the solid curve reflect Eq. (5). In all six rivers, data from the first group were plotted closely around the solid curve confirming the reasonably well predictability of Eq. (5) for transport capacities (Fig. 5). Data from the second group were significantly below the curve (Fig. 5) suggesting that bed-load transport rates associated with these data were below the capacities. This indicates that Eq. (2) is a robust criterion for determining whether a flow is transporting bed load at capacity or not. Data from the third group were further below the curve (Fig. 5) confirming the expected result that the associated flows transported bed load well below the capacities for the same θ values.

3.3. The breakout of the armor layer

Since data from the third group have D_{50} much less than D_{sub50}, rivers with the associated flows must have an armor layer, which significantly reduced the bed-load transport rates. The impact of the armor layer on bed-load transport has been characterized by an abrupt increase of bed-load transport rates at certain values of water discharge (Q) in plots of transport rates against Q (Emmett, 1976; Emmett and Wolman, 2001; Jackson and Beschta, 1982). At the stage before the sudden change, bed-load transport rates are relatively low and bed-load grains are primarily comprised of sand and fine gravels either from the bed or upstream. This stage has been termed as phase I. Once water discharge is above a threshold value, bed-load transport rate is greatly increased and bed-load grains contain more coarser materials due to the "breakup" of the armor bed. This stage has been termed as phase II. The two-phase model provides a theoretical framework characterizing the dependency of bed-load transport on bed

Table 2
Ranges of relevant hydraulic and sedimentary variables for data collected from the six gravel-bed rivers in Idaho, USA. In each parenthesis, the first number denotes the data from the first group and the second one reflects the data from the second group.

Sources	Middle Fork Simon River	Simon River near Shoup	Big Wood River	Rapid River	Salmon River Below Yankee Fork	Boise River	Total
Number of field data	6 (1 + 5)	11 (7 + 4)	3 (3 + 0)	4 (1 + 3)	5 (1 + 4)	2 (1 + 1)	29 (14 + 15)
Re ($\times 10^6$)	1.58–3.29	2.60–3.85	0.31–0.46	0.28–1.08	1.67–2.36	1.77–2.48	0.31–3.85
Re_*	40,242–60,006	27,326–58,278	44,055–49,989	30,680–64,716	13,377–60,190	27,732–57,579	27,326–60,190
Fr	0.63–0.72	0.48–0.54	0.57–0.60	0.40–0.69	0.40–0.47	0.53–0.54	0.48–0.72
D_{50} ($\times 10^{-3}$ m)	25–44	21–49	34–41	16.1–37.2	8–38	19–36	19–49
D_{sub50} ($\times 10^{-3}$ m)	36	28	25	23	21	23	21–36
θ	0.068–0.178	0.047–0.132	0.058–0.087	0.053–0.091	0.097–0.319	0.113–0.173	0.047–0.319
θ_c				0.06	0.06		
B	0.0006–0.014	0.003–0.052	0.00025–0.00037	0.00011–0.00694	0.00047–0.0078	0.002–0.020	0.00025–0.052

material components. However, no consistent method is available to determine the threshold value of Q that distinguishes the two phases. Using the piecewise regression analysis, Ryan et al. (2002) discovered that the critical Q is about 80% of the bankfull discharge. Bathurst (2005) developed a set of equations linking the critical Q to both channel bed slope and the degree of bed armoring. Instead of using Q, Lisle and Smith (2003) presented in a qualitative way the boundary between the two phases as the inflection point of a curve in the plot of bed-load transport rates versus dimensionless shear stress θ, which is scaled by the median size of bed surface grains D_{s50} rather than D_{50}.

Inasmuch as Eq. (5) characterizes bed-load transport capacities in gravel-bed rivers both with and without the armor layer, it can be used as an alternative approach to develop a quantitative criterion that identifies the hydraulic condition under which the armor layer breaks out. Before the breakup of the armor layer, the increase of flow may increase the possibility of grain exchange between bed load and bed surface grains (e.g., Wilcock and

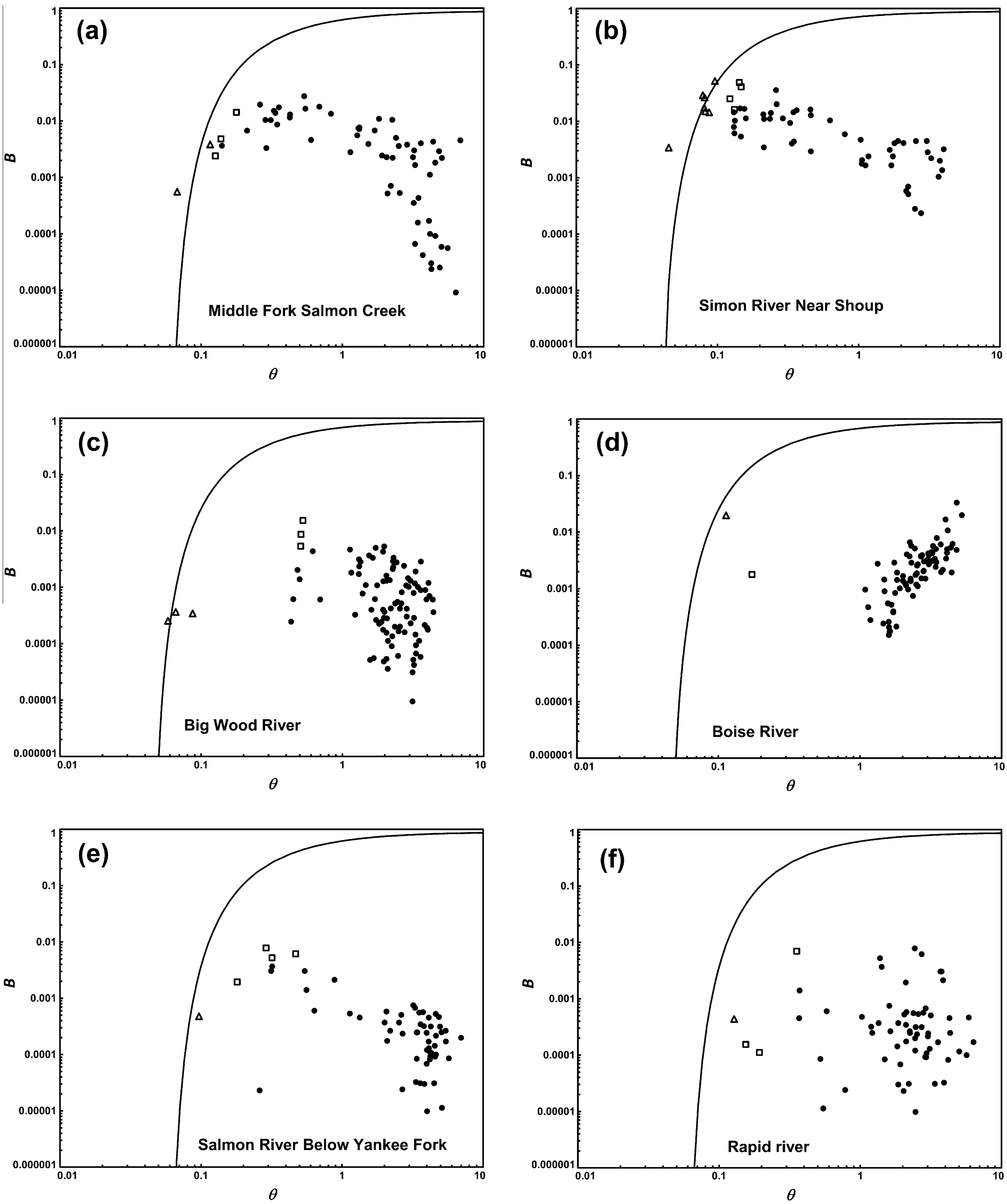

Fig. 5. Validation of both Eqs. (2) and (5) using the data from six gravel-bed rivers in Idaho. The open triangles denote the data from the first group, the open squares reflect those from the second group, the solid dots denote those from the third group, and the solid curve represents Eq. (5).

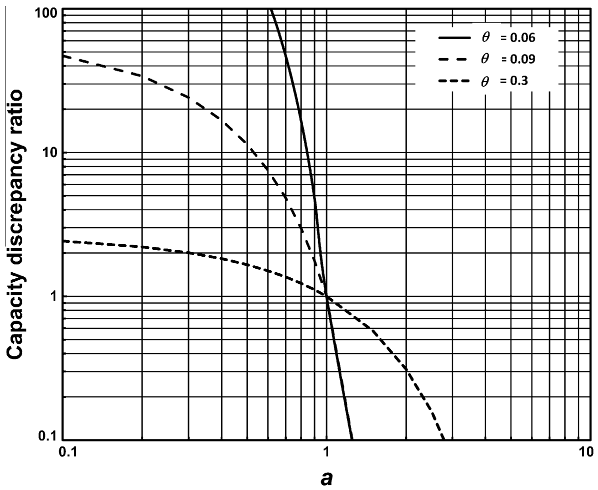

Fig. 6. The impact of errors in determining θ_c on the prediction of bed-load transport capacities.

DeTemple, 2005). Thus, channel bed serves as an additional source of sediment to bed load. The increase rate of bed-load transport rates with the increase of flow is high. After the armor layer disappeared, the increase rate is purely controlled by river hydraulics, which is relatively low. So, the transition between the two conditions can be generally characterized by the inflection point of Eq. (5), which may be quantitatively expressed as:

$$\theta = 3.5\theta_c \tag{6}$$

Eq. (6) shows that the armor layer in gravel-bed rivers should disappear when the θ value is 3.5 times that of θ_c, if bed load is transported at capacity.

3.4. Sensitivity of the θ_c value

The uncertainty involved in determining θ_c may have a significant impact on the calculation of the transport capacity using Eq. (5). For example, in Simon River near Shoup (Fig. 5), the open triangular with the lowest θ value has the bed-load transport capacity greater than that predicted by Eq. (5). Generally, values of θ_c in gravel-bed rivers could vary in a wide range (e.g., Martin, 2003; Martin and Ham, 2005). The complex mechanisms of sediment entrainment make it even harder to determine θ_c values (Diplas et al., 2008; Garcia et al., 2007). Therefore, none of the existing methods for determining θ_c values is prefect (Buffington and Montgomery, 1997). The extrapolation method employed here inevitably introduced errors in estimating the transport capacity. These possible errors were examined using the following sensitivity analysis.

For any "true" θ_c value, the error involved in the determination of θ_c may be expressed as $a\theta_c$ where a is a coefficient quantifying the degree of the error. If a is less than 1, then θ_c is underestimated, whereas if a is greater than 1, then θ_c is overestimated. For a given θ value and the "true" θ_c value, each designed value of a (representing the degree of the error involved in determination of θ_c) will lead to a transport capacity in terms of Eq. (5). The one associated with $a = 1$ represents the "true" transport capacity. The ratio of any other transport capacities calculated based on different values of a to the "true" capacity is termed as capacity discrepancy ratio. For a given "true" θ_c value, different θ values will be associated with different sets of the capacity discrepancy ratio. Fig. 6 showed an example for $\theta_c = 0.045$. As θ is very close to θ_c (i.e., $\theta = 0.06$), transport capacity is very sensitive to errors in θ_c. As θ increases, the transport capacity becomes less sensitive to the errors. When θ value is high, the transport capacity is not very sensitive to the errors in determining θ_c. Thus, the uncertainties in the determination of θ_c mainly have significant effect on the transport capacity when θ is close to θ_c. Given that transport capacity often occurs during high flows which tend to result in relatively high θ values, the uncertainty of determining θ_c should not significant affect the determination of the transport capacity in practice.

3.5. Limitations and application of Eq. (5)

Eq. (5) is not for predicting bed-load transport rates in gravel-bed rivers. Instead, it is developed for determining bed-load transport capacities for heterogeneous grains in gravel-bed rivers with turbulent, fully rough flows over approximately plane beds. In rivers with significant bed forms such as gravel bars and riffles, the calculated shear stress needs to be separated into two components, one is due to bed forms and the other is due to bed surface grains (e.g., Vanoni, 1975; Wiberg and Smith, 1989). The latter should be used in Eq. (5) for predicting bed-load transport capacities in these rivers. Eq. (5) is not affected by vegetation on channel banks. However, in channels with narrow width (e.g., the width/depth is less than 10), bank vegetation can cause significant side-wall drag effect. Hydraulic variables either flow depth or energy slope needs to be corrected before the calculation of θ (Vanoni and Brooks, 1957; Williams, 1970). Eq. (5) should be used in the relatively straight reaches of a meander river. Because Eq. (5) is developed and validated using the bed-load data based on bed-load samples collected during short-time periods, it can be used for predicting bed-load transport capacities during short-time periods. Also because Eq. (2) reflects the equilibrium condition during a long time period, Eq. (5) can give rise to transport capacities of gravel-bed rivers in equilibrium for a long time period. By using the median grain size of the bed substrate (D_{sub50}), the dominant discharge, and the representative channel slope, the transport capacity calculated from Eq. (5) is the bed-load transport rate of an equilibrium (i.e., graded) gravel-bed river in a long time period. Therefore, without dealing with the difficult task of determining all components of flow resistance (Eaton and Church, 2004), Eq. (5) can be used as a simple extreme hypothesis (Eaton et al., 2004; Knighton, 1998) to close the rational regime.

Given that the transport capacities predicted by Eq. (5) for low flows may be affected by the uncertainties involved in the calculation of θ_c, probably the most useful application of Eq. (5) is quick estimation, at the first approximation, of the "worst" sediment condition (i.e., bed-load is transported at capacity) during a potential big event. When the flow rate is very high (i.e., $\theta \gg \theta_c$), $G \approx 1$. Consequently, Eq. (5) is simplified as

$$B = 0.9 \text{ or } i_b = 0.9\omega \tag{7}$$

Eq. (7) means that the bed-load transport capacities of natural gravel-bed rivers during big events may be estimated by calculating the possible values of stream power, ω, which may be easily obtained using often available hydrologic and channel morphological information. For example, for a gravel-bed river with an USGS gauging station, the energy slope (S) can be roughly represented by the channel water surface slope, which may be obtained by surveying, and the discharge that may cause a flood (assuming equivalent to the discharge with the 50-year reoccurrence interval, Q_{50}) can be easily calculated using the recorded discharge data from that station in terms of flood frequency analysis. Thus, the corresponding stream power ω can be calculated by $\omega = Q_{50}S$. It follows that the bed-load transport capacity for this potential flood can be estimated as 0.9ω from Eq. (7).

Although this estimation is only a first approximation, it can be done quickly and should be practically useful for estimating the influence of bed load on flow depth by calculating flow resistance due to bed-load transport (Gao and Abrahams, 2004; Song et al., 1998). For a gravel augmentation project aiming at generating a desired fish habitat in a gravel-bed river reach, Eq. (7) could be used by the project manager to quickly estimate the amount of required gravels to sustain the project. If global warming leads to extreme events, gravel-bed rivers may be subject to increased flooding and sediment transport. Eq. (7) serves as a simple tool to provide the first approximation of the transport capacities for various emergence river management practices.

4. Conclusions

Bed-load transport capacity for heterogeneous grains in natural gravel-bed rivers was defined as the maximum possible transport rate a gravel-bed river can have for a given value of θ calculated using the median size of bed-load grains D_{50}. A simple, empirical equation for estimating the transport capacities was developed using the compiled data representing bed-load transport capacities in both low flows with an armor layer and high flows without it

$$B = 0.9G^6 \tag{5}$$

where $B = i_b/\omega$, i_b is the bed-load transport rate at capacity (kg m^{-1} s^{-1}), ω is the unit stream power per unit bed area (kg m^{-1} s^{-1}), $G = 1 - \theta_c/\theta$, θ and θ_c are dimensionless shear stress and the associated threshold value for the initial motion of grains, respectively. The data that represent bed-load transport capacities for heterogeneous grains can be identified by the criterion

$$D_{50} \geqslant D_{sub50} \tag{2}$$

where D_{sub50} is the median sizes of grains in the bed substrate. Errors in determining θ_c may affect the predicted bed-load transport capacities when θ is close to θ_c. For high θ values, which normally require peak discharges that can cause flooding, the value of θ_c does not significantly affect the predicted transport capacity. Thus, in practice, Eq. (5) can be used to assess the maximum possible bed-load transport rates if a given flood discharge would arrive. Such assessment can be very useful for various river management practices such as flood control and gravel augmentation for generating fish habitat.

Acknowledgements

The author thanks Don Siegel for reorganizing the structure of the paper for general clarity. The author also thanks the Associate Editor, Jonathan Laronne, and two other anonymous reviewers for providing the useful comments and suggestions.

References

Abrahams, A.D., Gao, P., 2006. A bed-load transport model for rough turbulent open-channel flows on plane beds. Earth Surface Processes and Landforms 31, 910–928.

Almedeij, J.H., Diplas, P., 2003. Bed-load transport in gravel-bed streams with unimodal sediment. Journal of Hydraulic Engineering 129 (11), 896–904.

Andrew, E.D., 1979. Scour and Fill in a Stream Channel, East Fork River, Western Wyoming, pp. 49.

Andrews, E.D., Parker, G., 1987. Formation of a coarse surface layer as the response to gravel mobility. In: Thorne, C.R., Bathurst, J.C., Hey, R.D. (Eds.), Sediment Transport in Gravel-Bed Rivers. John Wiley, New York, pp. 269–325.

Ashworth, P.J., Ferguson, R.I., 1989. Size-selective entrainment of bed load in gravel bed streams. Water Resources Research 25 (4), 627–634.

Barry, J.J., Buffington, J.M., King, J.G., 2004. A general power equation for predicting bed load transport rates in gravel bed rivers. Water Resources Research 40, W10401.

Bathurst, J.C., 2005. Effect of coarse surface layer on bed-load transport. Journal of Hydraulic Engineering 133 (11), 1192–1205.

Bathurst, J.C., Graf, W.H., Cao, H.H., 1987. Bed load discharge equations for steep mountain rivers. In: Thorne, C.R., Bathurst, J.C., Hay, R.D. (Eds.), Sediment Transport in Gravel-Bed Rivers. John Wiley & Sons, Chischester, pp. 453–492.

Bravo-Espinosa, M., 1999. Prediction of Bed-Load Discharge for Alluvial Channels. The University of Azriona, Tuson, Arizona. 276 pp.

Bravo-Espinosa, M., Osterkamp, W.R., Lopes, V.L., 2003. Bed-load transport in alluvial channels. Journal of Hydraulic Engineering 129 (10), 783–795.

Bridge, J.S., 2003. Rivers and Floodplains – Forms, Processes, and Sedimentary Record. Blackwell Science, Oxford.

Buffington, J.M., Montgomery, D.R., 1997. A systematic analysis of eight decades of incipient motion studies, with spatial reference to gravel-bedded rivers. Water Resources Research 33, 1993–2029.

Buffington, J.M., Montgomery, D.R., 1999. Effects of sediment supply on surface textures of gravel-bed rivers. Water Resources Research 35 (11), 3523–3531.

Bunte, K., Abt, S.R., Potyondy, J.P., Swingle, K.W., 2008. A comparison of coarse bed-load transport measured with bed-load traps and Helley–Smith samplers. Geodinamica Acta 21 (1–2), 53–66.

Clayton, J.A., Pitlick, J., 2008. Persistence of the surface texture of a gravel-bed river during a large flood. Earth Surface Processes and Landforms 33, 661–673.

Dietrich, W.E., Kirchner, J.W., Ikeda, H., Iseya, F., 1989. Sediment supply and the development of the coarse surface layer in gravel-bedded rivers. Nature 340 (20), 215–217.

Diplas, P. et al., 2008. The role of impulse on the initiation of particle movement under turbulent flow conditions. Science 322 (5902), 717–720.

Eaton, B.C., Church, M., 2004. A graded stream response relation for bed load-deominated streams. Journal of Geophysical Research 109, F03011.

Eaton, B.C., Church, M., Millar, R.G., 2004. Rational regime model of alluvial channel morphology and response. Earth Surface Processes and Landforms 29, 511–529.

Egiazaroff, I.V., 1965. Calculation of nonuniform sediment transportations. Journal of Hydraulic Division, American Society of Civil Engineers 91, 225–247.

Einstein, H.A., 1950. The Bed Load Function for Sediment Transportation in Open Channel Flows. United States Department of Agriculture, Soil Conservation Service, Washington, DC.

Einstein, H.A., Chien, N., 1953. Transport of Sediment Mixtures with Large Ranges of Grain Sizes. US Army Corps of Engineers, Missouri River Division, Omaha.

Emmett, W.W., 1976. Bed-load transport in two, large gravel-bed rivers, Idaho and Washington. In: Committee, S. (Ed.), Third Inter-Agency Sedimentation Conference. Water Resources Council, Denver, pp. 100–113.

Emmett, W.W., Wolman, M.G., 2001. Effective discharge and gravel-bed rivers. Earth Surface Processes and Landforms 26, 1369–1380.

Ferguson, R.I., 2003. The missing dimension: effects of lateral variation on 1-D calculations of fluvial bed-load transport. Geomorphology 56, 1–14.

Fernandez Luque, R., van Beek, R., 1976. Erosion and transport of bed-load sediment. Journal of Hydraulic Research 14, 127–144.

Gao, P., 2008. Transition between two bed-load transport regimes: saltation and sheet flow. Journal of Hydraulic Engineering 134 (3), 340–349.

Gao, P., Abrahams, A.D., 2004. Bed-load transport resistance in rough open-channel flows. Earth Surface Processes and Landforms 29, 423–435.

Garcia, C. et al., 2007. Processes of initiation of motion leading to bed-load transport in gravel-bed rivers. Geophysical Research Letters 34 (6), 4.

Gilbert, G.K., 1914. The Transportation of Debris by Running Water. US Geological Survey Professional Paper.

Gomez, B., 1983. Temporal variations in bed-load transport rates: the effect of progressive bed armouring. Earth Surface Processes and Landforms 8, 41–54.

Gomez, B., 2006. The potential rate of bed-load transport. In: Proceedings of the National Academy of Sciences of the United States of America, pp. 17170–17173.

Gomez, B., Church, M., 1988. A Catalogue of Equilibrium Bed-Load Transport Data for Coarse Sand Gravel-Bed Channels. Department of Geography, University of British Columbia.

Gomez, B., Church, M., 1989. An assessment of bed load sediment transport formulae for gravel bed rivers. Water Resources Research 25 (6), 1161–1186.

Hicks, D.M., Gomez, B., 2003. Sediment transport. In: Kondolf, G.M., Piegay, H. (Eds.), Tools in Fluvial Geomorphology. John Wiley & Sons Ltd., Chichester, pp. 425–462.

Jackson, W.L., Beschta, R.L., 1982. A model of two-phase bed-load transport in an oregon coast range stream. Earth Surface Processes and Landforms 7, 517–527.

Knighton, D., 1998. Fluvial Forms and Processes: A New Perspective. Arnold, London, 383 pp.

Laronne, J.B., Reid, I., Yitshak, Y., Frostick, L.E., 1994. The non-layering of gravel streambeds under ephemeral flood regimes. Journal of Hydrology 159, 353–363.

Lenzi, M.A., D'Agostino, V., Billi, P., 1999. Bed-load transport in the instrumented catchment of the Rio Cordon, part I: analysis of bed-load records, conditions and threshold of bed-load entrainment. Catena 36, 171–190.

Lisle, T.E., 2007. The evolution of sediment waves influenced by varying transport capacity in heterogeneous rivers In: Habersack, H., Piegay, H., Rinaldi, M. (Eds.), Gravel-Bed Rivers VI: From Process Understanding to River Restoration. Elsevier, Amsterdam, pp. 443–472.

Lisle, T.E., Madej, M.A., 1992. Spatial variation in armouring in a channel with high sediment supply. In: Billi, P., Hey, R.D., Thorne, C.R., Tacconi, P. (Eds.), Dynamics of Gravel-Bed Rivers. John Wiley and Sons, New York, pp. 277–296.

Lisle, T.E., Smith, B., 2003. Dynamic transport capacity in gravel-bed river systems. In: Araya, T., Kuroki, M., Marutani, T. (Eds.), International Workshop for Source to Sink Sedimentary Dynamics in Catchment Scale. Organizing Committee of the International Workshop for Sedimentary Dynamics, Sapporo, Japan.

Mackin, J.H., 1948. Concept of the graded river. The Geological Society of America Bulletin (59), 463–512.

Martin, Y., 2003. Evaluation of bed load transport formulae using field evidence from the Vedder River, British Columbia. Geomorphology 53, 75–95.

Martin, Y., Ham, D., 2005. Testing bed-load transport formulae using morphologic transport estimates and field data: lower Fraser River, British Columbia. Earth Surface Processes and Landforms 30, 1265–1282.

Meyer-Peter, E., Mueller, R., 1948. Formulas for bed load transport. In: Proceedings of the Second Meeting, IAHR, pp. 39–64.
Montgomery, D.R., Panfil, M.S., Hayes, S.K., 2000. Channel-bed mobility response to extreme sediment loading at Mount Pinatubo. Geology 27 (3), 271–274.
Parker, G., 1990. Surface-based bed-load transport relation for gravel rivers. Journal of Hydraulic Research 28 (4), 417–436.
Parker, G., 2006. Transport of gravel and sediment mixtures. In: Garcia, M.H. (Ed.), Sedimentation Engineering: Processes, Measurements, Modeling, and Practice. American Society of Civil Engineers, Reston, pp. 165–251.
Parker, G., Klingeman, P.C., 1982. On why gravel bed streams are paved. Water Resources Research 18 (5), 1409–1423.
Parker, G., Toro-Escobar, C.M., 2002. Equal mobility of gravel in streams: the remains of the day. Water Resources Research 38 (11), 1264–1271.
Parker, G., Klingeman, P.C., McLean, D.L., 1982. Bed-load and size distribution in paved gravel-bed streams. Journal of Hydraulic Division. American Society of Civil Engineers 108 (HY4), 544–571.
Powell, D.M., Reid, I., Laronne, J.B., 1999. Hydraulic interpretation of cross-stream variations in bed-load transport. Journal of Hydraulic Engineering 125 (12), 1243–1252.
Powell, D.M., Reid, I., Laronne, J.B., 2001. Evolution of bed-load grain-size distribution with increasing flow strength and the effect of flow duration on the caliber of bed-load sediment yield in ephemeral gravel-bed rivers. Water Resources Research 34 (1), 1463–1474.
Powell, D.M., Brazier, R., Parsons, A., Wainwright, J., Nichols, M., 2006. Sediment transfer and storage in dryland headwater streams. Geomorphology 88 (1), 152–166.
Recking, A., 2010. A comparison between flume and field bed load transport data and consequences for surface-based bed load transport prediction. Water Resources Research 46, W03518. doi:10.1029/2009WR008007.
Reid, L.M., Dunne, T., 1996. Rapid Evaluation of Sediment Budgets. Catena Verlag, Reiskirchen. 164 pp.
Reid, I., Laronne, J.B., Powell, D.M., 1995. The Nahal Yatir bed-load database: sediment dynamics in a gravel-bed ephemeral stream. Earth Surface Processes and Landforms 20, 845–857.
Reid, I., Powell, D.M., Laronne, J.B., 1996. Prediction of bed-load transport by desert flash floods. Journal of Hydraulic Engineering 122 (3), 170–173.
Ryan, S.E., Porth, L.S., Troendle, C.A., 2002. Defining phases of bed-load transport using piecewise regression. Earth Surface Processes and Landforms 27 (9), 971–990.
Sear, D.A., 1996. Sediment transport processes in pool-riffle sequences. Earth Surface Processes and Landforms 21, 241–262.
Simons, D.B., Senturk, F., 1992. Sediment Transport Technology: Water and Sediment Dynamics. Water Resources Publications, Littleton. 897 pp.
Song, T., Chiew, Y.M., Chin, C.O., 1998. Effect of bed-load movement of flow friction factor. Journal of Hydraulic Engineering 124 (2), 165–175.
Sutherland, A.J., 1987. Static armour layers by selective erosion. In: Thorne, C.R., Bathurst, J.C., Hey, R.D. (Eds.), Sediment Transport in Gravel-Bed Rivers. John Wiley & Sons, Chichester, pp. 243–267.
van Rijn, L.C., 1984. Sediment transport, part I: bed load transport. Journal of Hydraulic Engineering 110 (10), 1431–1456.
Vanoni, V.A., 1975. Sedimentation Engineering. ASCE, New York.
Vanoni, V.A., Brooks, N.H., 1957. Laboratory Studies of the Roughness and Suspended Load of Alluvial Streams, California Institute of Technology Sedimentation Laboratory, Report, E-86.
Wathen, S.J., Ferguson, R.I., Hoey, T.B., Werrity, A., 1995. Unequal mobility of gravel and sand in weakly bimodal river sediments. Water Resources Research 31, 2087–2096.
Wiberg, P.L., Smith, J.D., 1989. Model for calculating bed load transport of sediment. Journal of Hydraulic Engineering 115 (1), 101–123.
Wilcock, P.R., 2001. The flow, the bed, and the transport: interaction in flume and field. In: Mosley, M.P. (Ed.), Gravel-Bed Rivers V. New Zealand Hydrological Society Inc, Wellington, pp. 183–220.
Wilcock, P.R., Crowe, J.C., 2003. Surface-based transport model for mixed-size sediment. Journal of Hydraulic Engineering 129 (2), 120–128.
Wilcock, P., DeTemple, B.T., 2005. Persistence of armor layers in gravel-bed streams. Geophysical Research Letters 32, L08402.
Wilcock, P.R., Kenworthy, S.T., 2002. A two-fraction model for the transport of sand/gravel mixtures. Water Resources Research 38 (10), 1194.
Wilcock, P.R., Southard, J.B., 1989. Bed load transport of mixed size sediment: fractional transport rates, bed forms, and the development of a coarse bed surface layer. Water Resources Research 25, 1629–1641.
Wilcock, P., Pitlick, J., Cui, Y., 2009. Sediment Transport Primer: Estimating Bed-Material Transport in Gravel-Bed Rivers. US Department of Agriculture, Forest Service, Rocky Mountain Research Station, Fort Collins, CO.
Williams, G.P., 1970. Flume Width and Water Depth Effects in Sediment-Transport Experiments. US Geological Survey Professional Paper, 562H.
Yalin, M.S., 1977. Mechanics of Sediment Transport. Pergmon Press, Oxford, England. 298 pp.